Letter of Intent for an experiment at the FnPB/SNS

pNab: a program of study of polarized neutron beta decay

Nab/pNab Collaborations

30 December 2020

Abstract: The Nab/pNab collaboration undertake a program of studies of free neutron beta decay that is aimed (1) to determine the ratio of the coupling constants in free neutron beta decay, $\lambda = g_{\mathrm{V}}/g_{\mathrm{A}}$, with unprecedented precision, (2) to contribute to a test of the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and (3) to search for non-Standard Model (SM) forms of weak interaction that manifest themselves as scalar and/or tensor interactions. For this purpose, a large, novel magneto-electrostatic spectrometer, the Nab spectrometer, has been developed, and is being used to determine the correlation coefficients in unpolarized neutron beta decay: a, and b. In pNab, the second step, the same spectrometer will be used with a polarized neutron beam to determine two more correlation coefficients, A, and B. The measurements performed in Nab and pNab will provide a robust dataset with unprecedented sensitivity to λ and to new physics, and serve as a needed systematic check to resolve discrepancies in determinations of λ. This Letter of Intent is an expression of commitment of its authors to form a collaboration with the goal of reaching the ultimate sensitivity to new physics at the Fundamental Neutron Physics Beamline at the Spallation Neutron Source. This will be accomplished by pNab, to be staged in the FnPB immediately following the completion of measurements with unpolarized neutrons with Nab. pNab will use most of the existing Nab apparatus, and will require only modest additional resources.

1. Scientific motivation

The measurement of observables in free neutron beta decay falls within the broader field of study of the basic properties and symmetries of the electroweak interaction at low energies. Although successful without parallel, the present standard model (SM) of elementary particles and their interactions is known to be incomplete. Additional particles and phenomena must exist. Questions regarding possible extensions of the SM are being simultaneously addressed at the high energy frontier, using particle colliders, and at the precision frontier at lower energies. Neutron beta decay contributes to a precision test of the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which provides one of the most sensitive tests of our understanding of the electroweak interaction of quarks.

The most precise test of the Unitarity of the CKM matrix is available for the first row of matrix elements:

$$|V_{\mathrm{ud}}|^2 + |V_{\mathrm{us}}|^2 + |V_{\mathrm{ub}}|^2 = 1 \tag{1}$$

As the CKM matrix describes the mixing of quark states, it must be a unitary matrix. A failure of the CKM Unitarity test signals new physics, e.g., the effects of additional exchange bosons (see, e.g., [1–3]), or the existence a fourth quark generation [4, 5]. Using an effective field theory (EFT) framework, Refs. [6, 7] show that this test is sensitive to physics beyond the reach of Large Hadron Collider (LHC).

The most precise determination of V_{ud} is presently available from the analysis of superallowed Fermi (SAF) beta decays. The $\mathcal{F}t$ values, representing the product: "phase space factor" × "(partial) half-life" × "nuclear structure and radiative corrections", are averaged for SAF decays of a number of nuclides, and are used to determine V_{ud} through

$$|V_{\mathrm{ud}}|^2 = \frac{2984.43\,\mathrm{s}}{\mathcal{F}t(1+\Delta_{\mathrm{R}}^{\mathrm{V}})} \tag{2}$$

The analysis of superallowed Fermi beta decays by J. Hardy, I. Towner is the one generally adopted. Their latest and final compilation update is given in Ref. [8]. In recent work, C.-Y. Seng, M. Gorchtein et al. [9,10] recalculated the most controversial part of the inner radiative correction $\Delta_{\mathrm{R}}^{\mathrm{V}}$ (the contribution of the γW box diagram). They used dispersion relations to connect it to data from neutrino-hadron-scattering. Not only is this computation more precise than the previously established one (Ref. [11]), but its central value is also shifted. The new $\Delta_{\mathrm{R}}^{\mathrm{V}}$ was confirmed in Ref. [12] (Refinement of [11]), Ref. [13] (based on a direct Lattice QCD calculation for pion decay; a Lattice QCD calculation for neutron or proton decay does not exist yet), and Ref. [14]. Furthermore, in said diagram γ and W may connect to different nuclei, and therefore affect nuclear structure-dependent radiative correction (commonly called δ_{NS}) that are part of $\mathcal{F}t$ [10,15]). The updated superallowed beta decay analysis, gives $V_{\mathrm{ud}} = 0.97373(31)$ [8]. We note that at this time the Particle Data Group (PDG) does not use the new nuclear structure-dependent radiative corrections which come with larger uncertainties, and gives $V_{\mathrm{ud}} = 0.97370(14)$ [16]). These recent theoretical advances underscore the need for experimental measurements providing systematic checks with different sensitivity to these theoretical corrections.

To test CKM unitarity, one needs to combine V_{ud} with determinations of V_{us} and V_{ub}. The contribution of V_{ub} is too small to register in the sum of squares in Eq. 1. The two most precise evaluations of V_{us}, from K_{l3} and K_{l2} decays, respectively, disagree with each other by about 2σ (see [16], which uses data from [17]). In [16] the PDG summarizes the situation in the statement: *"One finds an overall 3 sigma deviation from unitarity. That deviation could be due a problem with $|V_{\mathrm{ud}}|$ theory (radiative corrections or nuclear physics), the lattice determination of $f_+(0)$ (the form factor for K_{l3} decays), or new physics."* The discrepancy can be reduced down to 1.7σ by including the revised nuclear structure-dependent radiative corrections [10,15], and selecting K_{l2} decays as the sole input for V_{us} [13] (it is 3.0σ using K_{l3} decays instead). Figure 1 illustrates the situation, where direct determinations from superallowed decays are contrasted with data from neutron beta decay, discussed next, mirror nuclei (not discussed here) and determinations of V_{us} that are combined with V_{ub} to extract V_{ud} using Eq. 1.

There is an opportunity for measurements of observables in free neutron beta decay to have an impact on this test. Basing the unitarity test on neutrons and/or pions is desirable due to the absence of nuclear structure-dependent corrections (δ_{C} and δ_{NS}). On the other hand, neutron

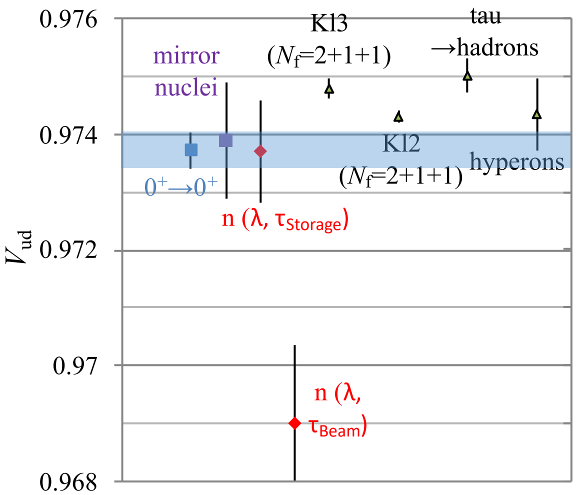

Figure 1: Determination of $V_{\rm ud}$. The blue square (and also the band) is the result from superallowed beta decays [8], the violet square is from nuclear mirror decays [14], and the red diamonds are from neutron beta decay. The green triangles are various determinations of $V_{\rm us}$ [16] that are combined with $V_{\rm ub}$ to extract $V_{\rm ud}$ using eq. 1.

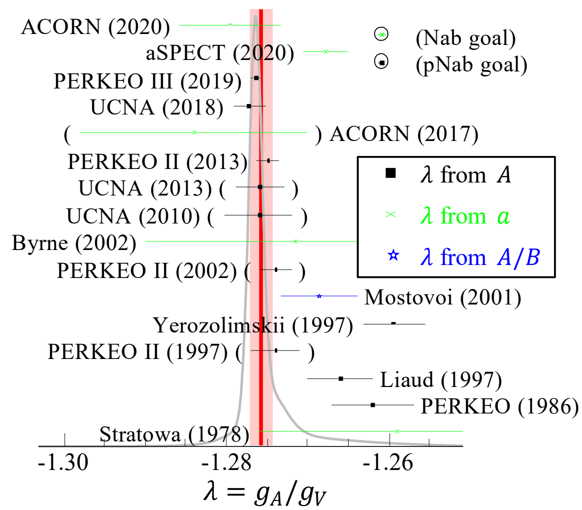

Figure 2: Summary of determinations of the ratio λ. The gray line is an ideogram. References for results shown are, from bottom to top, [27–42]. The red line shows the latest PDG average, evaluated before the aSPECT result publication and, excluding the new aCORN result.

beta decay shares the sensitivity to the inner radiative corrections with nuclear decays, including the superallowed (see [14] for the distinction between $\Delta_{\rm R}^{\rm V}$ and $\Delta_{\rm R}^{\rm A}$). $V_{\rm ud}$ can be determined using neutron beta decay data by measuring$\tau_{\rm n}$, the neutron lifetime, and $\lambda = g_{\rm A}/g_{\rm V}$, the ratio of the Gamow-Teller and Fermi coupling constants:

$$|V_{\rm ud}|^2 = \frac{5024.7\,{\rm s}}{\tau_{\rm n}(1+3\lambda^2)(1+\Delta_{\rm R}^{\rm V})}\,. \tag{3}$$

The PDG [16] gives the current average lifetime $\tau_{\rm n} = 879.4(6)\,{\rm s}$, but notes a long-standing disagreement between in-beam and stored neutron results. To be competitive, the neutron community must reduce the uncertainty in the neutron lifetime to $\tau_{\rm n} < 0.4\,{\rm s}$. On the other hand, measurements of correlation coefficients in neutron beta decays are used to determine λ. Matching SAF decay precision requires $\Delta\lambda/\lambda < 3.5 \cdot 10^{-4}$. The PDG averages existing experimental results to $\lambda = -1.2756(13)$ [16].* Analysis of neutron beta decay gives a value for $V_{\rm ud}$ consistent with the one from superallowed beta decay, but with lesser accuracy. There are multiple neutron lifetime experiments under construction that aim to at least reach an uncertainty of 0.4 s, and to resolve the disagreement between the results of two different lifetime measurement techniques [18]. In the US, these are UCNTau/UCNTau+ collaborations [19], BL2/3 at NIST [20], and a dedicated experiment

*We note that there is an inconsistency between λ given in the neutron properties section, and in the review on Unitarity of the CKM matrix (top row). Using the value in the latter review would bring the uncertainty of the neutron-based $V_{\rm ud}$ to be only 50% larger than the one from SAF beta decays in [8].

to understand the disagreement, UCNProBe [21].

The triple differential decay rate in neutron beta decay, at leading order [22] has the form:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_{\mathrm{e}}\mathrm{d}\Omega_{\mathrm{e}}\mathrm{d}\Omega_{\nu}} \propto \rho(E_{\mathrm{e}})(g_{\mathrm{V}}^2 + 3g_{\mathrm{A}}^2) \cdot \left[1 + a\frac{\vec{p}_{\mathrm{e}} \cdot \vec{p}_{\nu}}{E_{\mathrm{e}}E_{\nu}} + b\frac{m_{\mathrm{e}}}{E_{\mathrm{e}}} + \vec{\sigma}_{\mathrm{n}} \cdot \left(A\frac{\vec{p}_{\mathrm{e}}}{E_{\mathrm{e}}} + B\frac{\vec{p}_{\nu}}{E_{\nu}} + D\frac{\vec{p}_{\mathrm{e}} \times \vec{p}_{\nu}}{E_{\mathrm{e}}E_{\nu}}\right)\right]. \quad (4)$$

Several experiments have measured, or intend to measure, the correlation coefficients a, b, A, B, and D. The quantities $\rho(E_{\mathrm{e}})$ and $\vec{\sigma}_{\mathrm{n}}$ denote the phase space factor as a function of the relativistic electron energy E_{e} and the neutron spin, respectively. If outgoing particle spins are detected, more terms appear. In the low-energy limit of the SM, neutron beta decay is described as a $V - A$ interaction. Here, the Fierz interference term b vanishes, as confirmed in recent neutron beta decay measurements [23, 24]. (Tighter limits are obtained from combined analysis of multiple beta decays — see, e.g., [25].) The D coefficient is T-violating, tiny, and is neglected below (see [26] for the most recent limit). The coefficients a, A, and B are not small. They depend on $\lambda = g_{\mathrm{A}}/g_{\mathrm{V}}$ through

$$a = \frac{1-\lambda^2}{1+3\lambda^2}, \quad A = -2\frac{\lambda^2+\lambda}{1+3\lambda^2}, \quad B = 2\frac{\lambda^2-\lambda}{1+3\lambda^2}. \quad (5)$$

The most precise measurements of λ come from the measurement of the beta asymmetry A or neutrino electron correlation coefficient a in neutron beta decay studies. The present status is shown in Fig. 2. There is unfortunately some tension between the latest results from aSPECT and PERKEO III that is not explainable with Beyond-Standard-Model physics within the limits from other experiments. Better precision is expected to be reached in the Nab experiments (Refs. [43–45]), and in PERC (Refs. [46, 47]). The Nab collaboration will determine the neutrino electron correlation coefficient a, about 8 times more accurate than previously achieved. The derived value for λ will be two times more precise than the present best result (PERKEO III), and just at the level desired in the discussion after Eq. (3) to be competitive with SAF beta decay. Our intention is after Nab to continue with pNab, where the goal will be to determine the beta and the proton asymmetry to determine the A and B coefficients, both to better than 10^{-3}. Results from pNab are expected to be slightly more precise in λ. It will also allow a more direct comparison with several results in Fig. 2 based on beta asymmetry measurements; there are particular similarities with the UCNA project, although pNab remains the only coincidence experiment (see Ref. [44] for a more detailed discussion). The outcome of this program is not only a test of the CKM Unitarity that avoids uncertainties due to nuclear corrections. The result can alternatively be interpreted as a test of the CVC hypothesis [48], and as a verification of the new radiative correction calculations.

The ratio λ can be calculated from first principles using Lattice QCD. Recent attempts [49, 50] reach a percent level accuracy. While impressive, this uncertainty is still too large to replace direct measurements. On the other hand, the comparison of calculated and measured λ is a sensitive tool to indicate right-handed currents [14, 51].

Additional tests of the Standard Model can be performed if the electron energy dependences of the correlation coefficients are analyzed; Ref. [52] provided a framework in which such an analysis can be performed to verify the weak magnetism term, or to restrict "second class" hadronic matrix elements.

2. Measurement of the beta and the proton asymmetries with the Nab spectrometer

Electron and proton asymmetries can be measured using the Nab spectrometer with minimal modifications; this is the goal of the pNab experiment. The basic setup shown in Fig. 3 will remain the same, with the addition of a neutron polarizer and some beamline modifications to make space for it. The electron asymmetry measurement will use a configuration with the electrostatic voltages set such that all protons are detected in the lower detector. Coincidence between electrons and protons from the same neutron decay is still required, along with its strong suppression of background-related uncertainties. The proton asymmetry will be measured with the upper detector serving as the proton detector, as in the measurement of a with Nab.

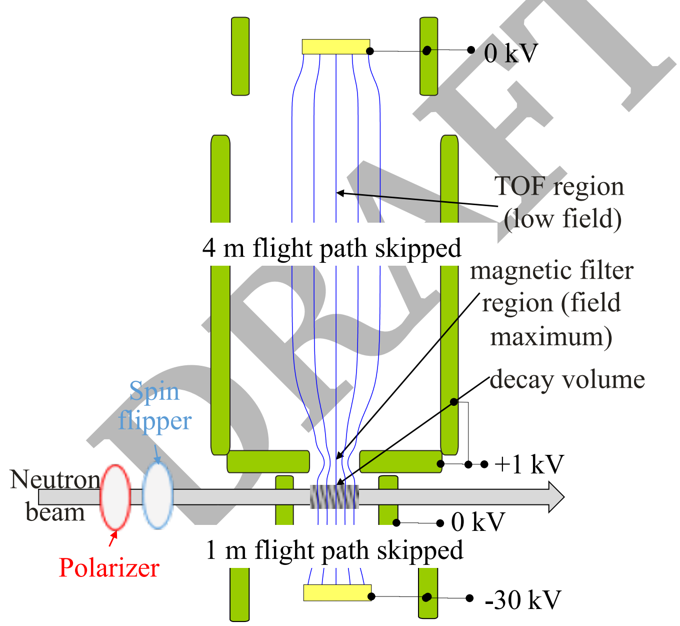

Figure 3: Principle of the pNab spectrometer. Magnetic field lines (shown in blue) electrodes (light green boxes), and coils (not shown) possess cylindrical symmetry around the vertical axis. The only difference compared to the Nab configuration is that the neutron beam will be polarized in pNab through the addition of a polarizer shown as the red oval. Voltages shown correspond to the beta asymmetry measurement configuration.

Both electron and proton asymmetries are of the type

$$\text{differential decay rate} \propto (1 + \alpha \cos\theta_0)\,, \tag{6}$$

where θ_0 is the initial angle of electron (or proton) momentum relative to the neutron beam polarization (i.e., the magnetic field) at the moment of the neutron decay, while α designates the size of the asymmetry, as follows. For the beta asymmetry, the differential decay rate is usually evaluated as a function of the electron energy, with $\alpha = A \cdot \beta_e$. For the proton asymmetry, $\alpha = C$ if proton energy dependence is not recorded.†

The most important sources for uncertainties in a measurement of the electron or proton asymmetry using the Nab spectrometer are discussed below.

2.1. *Statistical uncertainty*

Table 1 presents the statistical sensitivity of the beta asymmetry A in a Standard Model fit ($b = 0$), and of the Fierz term b (with A as another free parameter). The likely value for the threshold for the electron kinetic energy, $E_{e,kin,min} = 100\,\mathrm{keV}$, together with the goal for the statistical uncertainty in $\Delta A/A = 8 \times 10^{-4}$, translates to a minimum of $N = 2.5 \times 10^9$ neutron decays in the fiducial volume. The statistical accuracy goal will be reached after about 9 calendar months. Then, we would have $|\Delta b| = 0.007$. After only 6 weeks of beam time, pNab would already get to $\Delta A/A = 1.5 \times 10^{-3}$. This is calculated at an expected decay rate of $400\,\mathrm{s}^{-1}$and a down time of the experiment of 50%. We note that the neutron beam flux for a polarized beam is lower than the flux of an unpolarized beam due to the polarizer, and depends on the method used to achieve beam polarization. The decay rate given above assumes a transmission of the polarizer of $T_n = 25\%$. Table 1 assumes perfect neutron beam polarization. We discuss in section 2.3 below a beam polarizer that comes close.

Table 1: Statistical uncertainty in the determination of A (in a SM fit), and b (in a fit to the electron asymmetry with A as a second fit parameter), and an electron energy threshold $E_{e,kin}$ as shown. N is the number of neutron decays in the fiducial volume.

lower $E_{e,kin}$ cutoff:	none	100 keV	200 keV
σ_A	$4.3/\sqrt{N}$	$4.8/\sqrt{N}$	$7.8/\sqrt{N}$
σ_b	$300/\sqrt{N}$	$350/\sqrt{N}$	$500/\sqrt{N}$

2.2. *Solid angle acceptance for proton and electron detection*

In a symmetric spectrometer such as PERKEO III or UCNA, the measurement precision relies on the fact that the accepted solid angle of each detector is a hemisphere, and the average angle of electron (proton) momentum with the neutron spin is given as $\overline{\cos\theta_0} \sim 1/2$ with a correction due to the magnetic mirror effect. The asymmetric spectrometer design eliminates the unwanted magnetic mirror effect, and replaces it by the requirement to determine the solid angle of the upper and lower detectors. The cutoff angle for each detector depends on the magnetic field at the position

†We note that there are different definitions in the literature; ours follows the most recent measurement [16, 53]

of neutron decay. The measured electron asymmetries in both detectors can be combined in such a way that the angle cutoff of each detector drops out in leading order, or in a different way that allows us to extract the solid angle of the upper detector *in situ* from the beta asymmetries in both detectors, e.g., for subsequent use in a proton asymmetry measurement. We show in the appendix of Ref. [44] that the magnetic field inhomogeneities can be neglected for the Nab setup. Therefore, no high precision magnetic field measurements are needed, and the systematic uncertainty due to the solid angle is negligible, as is the uncertainty due to the imperfect knowledge of the neutron beam position. We note that the usual arrangement with two identical detectors used for electron and proton asymmetry measurements is replaced by two different detectors, but with a difference that is precisely understood.

2.3. Neutron beam polarization

A critical point in these measurements will be the precision of the knowledge of the neutron beam polarization. The earlier SNS proposal called for polarizing the neutron beam with a cell containing polarized Helium-3 (see Ref. [56, 57]), and relying on the known time-dependence of the polarization as a tool to analyze it. The advantage is that this provides an in-situ measurement of the degree of polarization. The potential issue is that for a reasonable transmission, the neutron beam polarization is low ($\sim 80\%$) which renders systematic errors hard to detect. An alternative method, which has been developed, tested, and is being used by the ILL group [58, 59], has the neutron beam polarized with crossed supermirrors to a very high degree ($P_{\mathrm{n}} > 99.7\%$), and analyzed with an opaque polarized Helium-3 spin filter. In this letter of intent, it is argued for a new device: a modern Solid State Polarizer [60, 61]. Figure 4 shows a sketch of the setup planned for FNPB (the optimization has to be tailored to the beam properties).

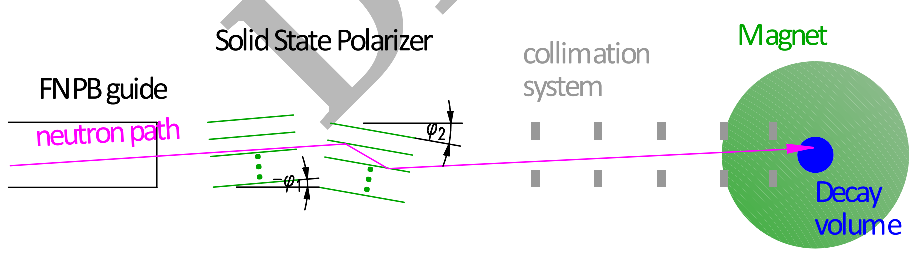

Figure 4: Proposed setup for studies of polarized neutron beta decay with pNab.

The solid state polarizer consists of two stacks of 180 μm thin sapphire plates, coated with modern supermirror and anti-reflective coating on both sides. Most neutrons undergo at least two reflections (and all at least one), guaranteeing a degree of polarization as large as for crossed supermirrors. At the same time, neutron transmission is much higher, as neutrons go on slightly attenuated through a few cm of sapphire substrate. In contrast, usual supermirror polarizers comprise a stack of thicker and much longer $(30-50\,\text{cm})$ glass substrates with gaps in between; the length renders the substrate opaque, leading to large neutron losses. Assembly will be done in a cleanroom to avoid dust that could limit the degree of parallelism of the sapphire plates. Simulations for the FNPB predict 99.5% polarization at 40% transmission (just behind the polarizer), which degrades to 99.5% polarization and about $20-25\%$ transmission in the decay volume of the experiment (the quoted range reflects the available choice of the supermirror quality. The step to 25% is pricey). This is similar to the recently built device at ILL (99.7% polarization at 33% transmission just behind polarizer [62], though the same performance cannot be expected at FNPB which has a higher divergence incoming beam. In other words, the Solid State Polarizer combines high polarization and transmission (even superior to a Helium-3 polarizer) with the capability to easily switch between polarized and unpolarized beam as for Helium-3, shows no time dependence of the polarization efficiency unlike Helium-3, and does not degrade statistical sensitivity of the experiment. In addition, it is now a proven technology.

2.4. Electron energy calibration

The pNab experiment plans to use the same detector system as the one used in Nab. The simulated electron energy response of the Nab detector system is shown in Fig. 5. Its width is substantially smaller than with a plastic scintillator detector. A set of radioactive calibration sources will be used to determine the detector response function, and to establish the linearity of the relationship between deposited energy and ADC channel. The sources are backed by thin, e.g., $10\,\mu\text{g/cm}^2$, carbon foils, and are movable within the fiducial volume so as to reach every point in the detector. Six possible candidates for such calibration sources have been used in Ref. [33]. Possible

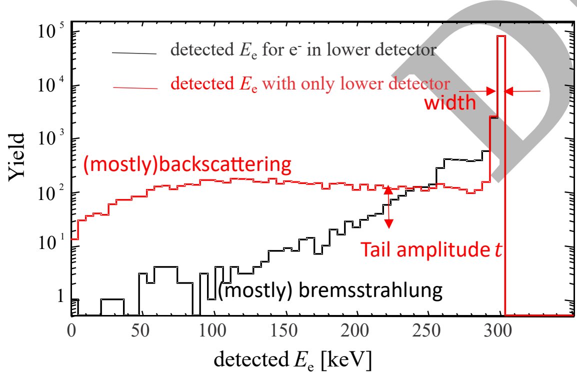

Figure 5: Simulated detector electron energy response for electrons incoming with $E_e = 200\,\text{keV}$. The red curve is for a single detector, and shows a large tail due to electron backscattering. The black curve is for both detectors. Backscattering is suppressed, and the remaining tail is mostly due to Bremsstrahlung.

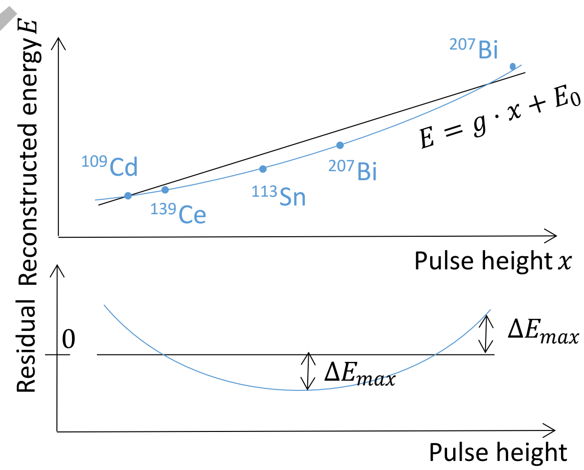

Figure 6: Relationship between energy deposited in a detector and the pulse height of the output signal. These synthetic data exaggerate possible types of nonlinearity.

deviations from perfect linearity of the relationship between pulse height and deposited energy are shown in Fig. 6. Table 2 shows the sensitivity of the beta asymmetry to various imperfections in the detector response. If the detector performs to the expectation brought forward for the measurements with Nab, the leading uncertainty in a beta asymmetry measurement with pNab will not be related to the detector.

Table 2: Detector-related uncertainties for the planned measurements with Nab and pNab. The meaning of the parameters are explained in Figs. 6 and 5.

Specification for	$\Delta a = 3 \times 10^{-5}$ (Nab)	$\Delta b = 10^{-3}$ (Nab)	$\Delta A = 3 \times 10^{-5}$ (pNab)
Gain factor ($\Delta g/g$)	fit parameter	fit parameter	0.0018
Offset E_0	0.3 keV	0.06 keV	0.2 keV
nonlinearity ($\lvert\Delta E_{\max}\rvert$)	1.5 keV	0.06 keV	0.3 keV
Tail to peak ratio (Δt)	0.01%	0.2%	2.4%

2.5. Electric field homogeneity

Protons from decays in a shallow electrostatic potential minimum with a momentum close to parallel to the magnetic field are trapped, which affects the solid angle of detector acceptance. A proton asymmetry measurement in a symmetric spectrometer is very difficult: Unwanted electrostatic potential variations need to be reduced to well below 1 mV, which is hard to achieve and even harder to verify. The asymmetric Nab spectrometer is much better suited for a proton asymmetry measurement: The filter in Nab allows only protons to pass whose momentum just after decay points mostly upwards. Protons which are trapped in the decay volume would not pass the filter even in the absence of the trap. In Nab, the leading contribution to the list of uncertainties from unwanted electrostatic potential variations is a potential difference between the filter region and the fiducial volume. A filter–fiducial volume potential difference as large as 100 mV changes the proton asymmetry by less than 0.01%, and can be neglected.

The beta asymmetry is much less sensitive to trapping, as electron energies are much higher.

3. Budget and schedule

The pNab experiment needs a modest investment of funds, as shown in Tab. 3.

The pNab experiment needs about 3 months to change over to a polarized beam and determine the degree of polarization with sufficient precision. As detailed in section 2.1, with only 6 weeks of data taking for pNab, the uncertainty of the result of pNab would surpass the one of the present best experiment, PERKEO III. And with 9 months of data taking, the goal of having a result that allows to test CKM Unitarity with the same precision as SAF, is reached. The latter assumes the expected improvement in the precision of the neutron lifetime.

A decision on pNab has to be made before Nab data taking is ending: Funds need to be available

Table 3: Budget estimate for pNab

Item	Budget [k$]
Solid State Polarizer	150
Si Detectors	100
Changes to beamline	100
Contingency	200
Total	550

early enough to start contracting and fabrication of the the longest lead-time item, which is the Solid State Polarizer. Early enough means: Summer 2022, which is 1.5 years before the start of pNab.

4. Summary

The pNab experiment promises a measurement of the beta asymmetry to substantially better than $\Delta A/A = 10^{-3}$, limited by statistics, and competitive with other current experiments. The main systematic uncertainties in this measurement are related to the detector, and pNab will have an important synergy with the Nab experiment in that the detector characterizations made for Nab are more than sufficient for pNab. Together, Nab and pNab measurements will provide a unique study of the CKM matrix unitarity, with very different systematics compared to other existing or planned measurements. CKM unitarity appears to be violated by about 3σ since new, more precise calculations of the inner radiative correction became available, making the proposed pNab extension to Nab well motivated.

The pNab experiment promises to measure the proton asymmetry, too, which has not been discussed here. Of particular interest is the planned determination of $|\Delta b_\nu| = 3 \times 10^{-3}$, probably through taking the systematically cleaner ratio of electron and proton asymmetry.

References

1. W. Marciano, A. Sirlin, Phys. Rev. D 35, 1672 (1987)
2. A. Kurylov, M. Ramsey-Musolf, Phys. Rev. Lett. 88, 071804 (2000)
3. S. Alioli et al., J. High Energ. Phys. 086 (2017)
4. W. Marciano, A. Sirlin, Phys. Rev. Lett. 56, 22 (1986)
5. P. Langacker, D. London, Phys. Rev. D 38, 886 (1988)
6. V. Cirigliano et al., Nucl. Phys. B 830, 95 (2010)
7. M. Gonzáles-Alonso et al., Prog. Nucl. Part. Phys. 104, 165 (2019)
8. J.C. Hardy, I.S. Towner, Phys. Rev. C 102, 045501 (2020)

9. C.-Y. Seng et al., Phys. Rev. Lett. 121, 241804 (2018)
10. C.-Y. Seng et al., Phys. Rev. D 100, 013001 (2019)
11. W.J. Marciano, A. Sirlin, Phys. Rev. Lett. 96, 032002 (2006)
12. A. Czarnecki et al., Phys. Rev. D 100, 073008 (2019)
13. X. Feng et al., Phys. Rev. Lett. 124, 192002 (2020)
14. L. Hayen, arXiv:2010.07262
15. M. Gorchtein, Phys. Rev. Lett. 123, 042503 (2019)
16. Particle Data Group, The Review of Particle Physics (2020), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) (Particularly relevant is the review "66. Vud, Vus, the Cabibbo Angle, and CKM Unitarity")
17. Flavor Lattice Averaging Group, FLAG Review 2019, arXiv:1902.08191
18. G. Greene, P. Geltenbort, Sci. Am. 314, 36 (2016)
19. R.W. Pattie Jr et al., Phys. J. Web Conf. 219, 03004 (2019)
20. F. Wietfeldt et al., Phys. Proc. 51, 54 (2014)
21. Z. Tang, APS April Meeting 2019, abstract id.H14.005
22. J.D. Jackson et al., Phys. Rev. 106, 517 (1957)
23. H. Saul et al., Phys. Rev. Lett. 125, 112501 (2020)
24. X. Sun et al., Phys. Rev. C 101, 035503 (2020)
25. R.W. Pattie, Jr. et al., Phys. Rev. C 88, 048501 (2013) and Phys. Rev. C 92, 069902(E) (2015)
26. T.E. Chupp et al., Phys. Rev. C 86, 035505 (2012)
27. Ch. Stratowa et al., Phys. Rev. D 18, 3970 (1978)
28. P. Bopp et al., Phys. Rev. Lett. 56, 919 (1986)
29. P. Liaud et al., Nucl. Phys. A 612, 53 (1997)
30. H. Abele et al., Phys. Lett. B 407, 212 (1997)
31. B. Yerozolimsky et al., Phys. Lett. B, 412 (1997)
32. Yu.A. Mostovoi et al., Phys. At. Nucl. 64, 1955 (2001)
33. H. Abele et al., Phys. Rev. Lett. 88, 211801 (2002)
34. J. Byrne et al., J. Phys. G 28, 3125 (2002)
35. J. Liu et al., Phys. Rev. Lett. 105, 181803 (2010)
36. M.P. Mendenhall et al., Phys. Rev. C 87, 032501(R) (2013)

37. D. Mund et al., Phys. Rev. Lett. 110, 172502 (2013)

38. G. Darius et al., Phys. Rev. Lett. 119, 042502 (2017)

39. M.A.-P. Brown et al., Phys. Rev. C 97, 035505 (2018)

40. B. Märkisch et al., Phys. Rev. Lett. 122, 242501 (2019)

41. M. Beck et al., Phys. Rev. C 101, 055506 (2020)

42. M.T. Hassan et al., submitted to Phys. Rev. C, arXiv:2012.14379

43. D. Počanić et al., Nucl. Inst. Meth. A 611, 211 (2009)

44. S. Baeßler et al., J. Phys. G 41, 114003 (2014)

45. J. Fry et al., Eur. Phys. J. Web Conf. 219, 04002 (2019)

46. D. Dubbers et al., Nucl. Instr. Meth. A 596, 238 (2008)

47. X. Wang et al., Eur. Phys. J. Web Conf. 219, 04007 (2019)

48. D. Dubbers, M.G. Schmidt, Rev. Mod. Phys. 83, 1111 (2011)

49. R. Gupta et al., Phys. Rev. D 98, 034503 (2018)

50. C.C. Chang et al., Nature 558, 91 (2018)

51. V. Cirigliano et al., Prog. Part. Nucl. Phys. 71, 93 (2013)

52. S. Gardner, B. Plaster, Phys. Rev. C 87, 065504 (2013)

53. M. Schumann et al., Phys. Rev. Lett. 100, 151801 (2008)

54. F. Glück, Phys. Lett. B 436, 25 (1998)

55. R. Alarcon et al., PANDA experiment proposal, available at `http://research.physics.lsa.umich.edu/chupp/panda/PANDA@SNS.pdf`, (2007)

56. R. Alarcon et al., abBA experiment proposal, available at `http://nab.phys.virginia.edu/ABBA_proposal_2007.pdf`, (2007)

57. S.I. Penttila, J.D. Bowman, J. Res. Natl. Inst. Stand. Technol. 110, 309 (2005)

58. M. Kreuz et al., Nucl. Inst. Meth. A 547, 583 (2005)

59. Ch. Klauser et al., Phys. Proc. 42, 99 (2013)

60. A.K. Petukhov et al., Nucl. Inst. Meth. A 838, 33 (2016)

61. A.K. Petukhov et al., Rev. Sci. Instr. 90, 085112 (2019)

62. A.K. Petoukhov et al., Contribution to ILL Annual report 2020, submitted