

# Study of neutron beta decay with the Nab experiment

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**Abstract.** The current three sigma tension in the unitarity test of the Cabibbo-Kobayashi-Maskawa (CKM) matrix is a notable problem with the Standard Model of elementary particle physics. A long-standing goal of the study of free neutron beta decay is to better determine the CKM element  $V_{ud}$  through measurements of the neutron lifetime and a decay correlation parameter. The Nab collaboration intends to measure  $a$ , the neutrino-electron correlation, with accuracy sufficient for a competitive evaluation of  $V_{ud}$  based on neutron decay data alone. This paper gives a status report and an outlook.

## 1 Introduction

Despite its unparalleled successes, the present Standard Model (SM) of elementary particles and their interactions is known to be incomplete. Additional particles and phenomena must

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exist. Questions regarding possible extensions of the SM are being simultaneously addressed at the high energy frontier, using particle colliders, and at the precision frontier, using small scale precision experiments. Neutron beta decay contributes to a precision test of the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, one of the most sensitive tests of our understanding of the electroweak interaction of quarks.

The most precise test of the unitarity of the CKM matrix is available for the first row:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \Delta \quad (1)$$

To test CKM unitarity, one determines  $V_{ud}$  in nuclear or neutron beta decay and  $V_{us}$  in certain Kaon decays. The contribution of  $V_{ub}$  is too small to register in Eq. (1) at the present level of precision. Current experiments reviewed below indicate  $\Delta \sim 10^{-3}$ , contrary to the SM expectation of  $\Delta = 0$ . A failure of the CKM unitarity test indicates new physics, e.g., the effects of additional exchange bosons (e.g., [1, 2]), anomalous couplings (e.g., [3, 4]), or the existence of a fourth quark generation [5, 6]. Refs. [4, 7–9] use an effective field theory (EFT) approach to show that this test is sensitive to physics with a reach comparable to that of the CERN Large Hadronic Collider, motivating intensive development of new analysis tools which integrate low energy constraints with those from collider measurements.

The most precise determination of  $V_{ud}$  is presently obtained from the analysis of superallowed Fermi (SAF) beta decays. The  $\mathcal{F}t$  values (the product of “phase space factor”, “(partial) half-life” and “nuclear structure and radiative corrections”) for multiple nuclides undergoing SAF decays are averaged, and are used to determine  $V_{ud}$  through

$$|V_{ud}|^2 = \frac{2984.43 \text{ s}}{\mathcal{F}t(1 + \Delta_R^V)} \quad (2)$$

Since 2018, the inner radiative correction  $\Delta_R^V$  has substantially shifted, and its dominant uncertainty (the contribution of the  $\gamma W$  box diagram) has been reduced in Refs. [10–14]. A preliminary lattice calculation [15] for this contribution in neutron beta decay gets a result similar to the new value for the inner radiative correction.

The analysis of  $\mathcal{F}t$  values in SAF decays in Ref. [16] is the one generally adopted. It constitutes a substantial update of previous work: It uses the revised inner radiative correction  $\Delta_R^V$ , and revised nuclear structure-dependent radiative corrections (commonly called  $\delta_{NS}$ ) that take into account the revised computation of the  $\gamma W$  box diagram [17]. The Particle Data Group (PDG) [18] recognizes the new input and gives as the recommended value from SAF decays  $V_{ud} = 0.97373(11)_{\text{exp.,nucl.}}(9)_{\text{RC}}(27)_{\text{NS}}$ .

There is an opportunity for free neutron beta decay to offer a competitive test of CKM unitarity with Eq. (1), and its potential precision similarly benefits from the work on the inner radiative correction. The extraction of  $V_{ud}$  from neutron and pion beta decay is not affected by nuclear corrections. The triple differential decay rate in neutron beta decay at leading order [19] — assuming  $T$ -invariance and no detection of spins of the final state particles — has the form

$$d^3\Gamma \propto \rho(E_e)G_F^2 V_{ud}^2 (1 + 3\lambda^2) \left( 1 + a \frac{\vec{p}_e \cdot \vec{p}_\nu}{E_e E_\nu} + b \frac{m_e}{E_e} + \sigma_n \cdot \left[ A \frac{\vec{p}_e}{E_e} + B \frac{\vec{p}_\nu}{E_\nu} \right] \right) d\Omega_e d\Omega_\nu dE_e \quad (3)$$

The quantities  $\rho(E_e)$  and  $\sigma_n$  denote the phase space factor as a function of the (relativistic) electron energy  $E_e$  and the neutron spin, respectively. Several experiments have measured, or intend to measure, the correlation coefficients  $a$ ,  $b$ ,  $A$  and  $B$ . At tree-level in the standard model and upon neglecting terms proportional to the small neutron recoil, the interaction is a pure  $V - A$  (vector minus axial vector) for which the Fierz interference term  $b$  vanishes

identically[20]. The coefficients  $a$ ,  $A$ , and  $B$  depend on  $\lambda = g_A/g_V$ , the ratio of the Gamow-Teller and Fermi coupling constants, through

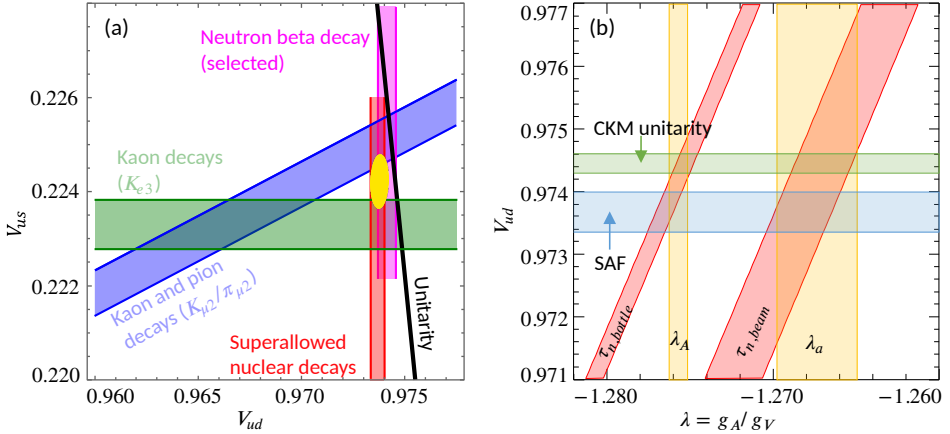
$$a = \frac{1 - \lambda^2}{1 + 3\lambda^2}; A = -2 \frac{\lambda^2 + \lambda}{1 + 3\lambda^2}; B = 2 \frac{\lambda^2 - \lambda}{1 + 3\lambda^2} . \quad (4)$$

The parameter  $\lambda$  is most precisely determined by the beta asymmetry  $A$  ( $dA/d\lambda = -0.37$ ) and the neutrino electron correlation coefficient  $a$  ( $da/d\lambda = -0.30$ ) in neutron beta decay studies. The neutrino asymmetry  $B$  ( $dB/d\lambda = -0.08$ ) is less sensitive to  $\lambda$ . The PDG averages existing experimental results to  $\lambda = -1.2754(13)$  with a scale factor of  $S = 2.7$ . Most of the data used is from measurements of the beta asymmetry.

The quantity  $V_{ud}$  is determined using neutron beta decay data by combining  $\tau_n$ , the neutron lifetime, and  $\lambda$ :

$$|V_{ud}|^2 = \frac{5024.7 \text{ s}}{\tau_n (1 + 3\lambda^2) (1 + \Delta_R^V)} \quad (5)$$

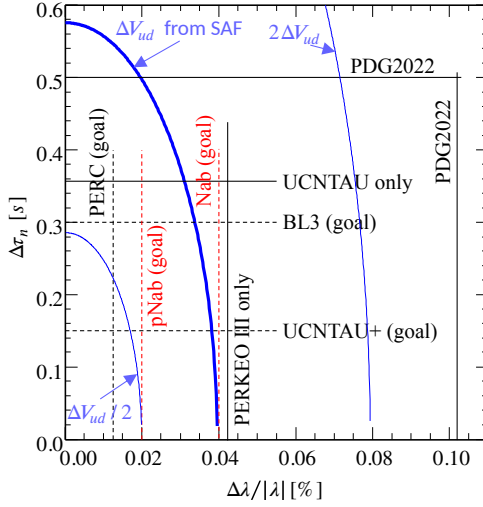
The PDG gives the current average lifetime  $\tau_n = 878.4(5) \text{ s}$ , but notes the long-standing disagreement between in-beam ( $\tau_{n,\text{beam}}$ ) and storage-bottle ( $\tau_{n,\text{bottle}}$ ) results.



**Figure 1.** (a) Combined analysis of  $V_{ud}$  from selected neutron beta decays and SAF nuclear decays,  $V_{us}/V_{ud}$  and  $V_{us}$  from kaon and pion decays. If unitarity holds, all  $1\sigma$  bands have to intersect the black line ( $|V_{ud}|^2 + |V_{us}|^2 = 1$ ) at the same point; here, unitarity is violated by  $2.8\sigma$  (see [21]). (b) Current  $1\sigma$  constraints for  $V_{ud}$  and  $\lambda$ . Red and yellow neutron beta decay bands are discussed in the text. Blue and green bands denote the  $V_{ud}$  values from SAF and from kaons through  $|V_{ud}|^2 = 1 - |V_{us}|^2$ , respectively.

Figure 1(a) shows a combined analysis of the CKM unitarity test, taken from Ref. [21]. Instead of the usual full world average, we have included only the most precise experimental data for  $\lambda$  from a measurement of  $A$  in Ref. [22] and  $\tau_n$  from Ref. [23] from a measurement in a neutron bottle, in evaluating the neutron beta decay limit. The  $V_{ud}$  values from SAF and neutrons are consistent. The figure also shows the conflicting limits from kaon decays. The yellow ellipse specifies the  $1\sigma$  contour of the region with the most likely values for  $V_{us}$  and  $V_{ud}$ . It misses unitarity by  $2.8\sigma$ . Inclusion of recent work to obtain  $V_{us}$  from tau decays [9] would increase the deviation.

Figure 1(b) allows a closer look at the current neutron beta decay data. The red  $1\sigma$  bands denote  $V_{ud}$  values obtained from neutron lifetimes measured by the bottle method ( $\tau_{n,\text{bottle}}$ , summarized in [18]) and beam method ( $\tau_{n,\text{beam}}$  [24, 25]), respectively. The yellow  $1\sigma$  bands denote values of the ratio  $\lambda$  obtained from the beta asymmetry  $A$  ( $\lambda_A$ , compiled in [18]) and



**Figure 2.** Uncertainty  $\Delta V_{ud}$  from neutron beta decay (Eq. (5)) as a function of uncertainties in the experimental input. Contours of constant values of  $\Delta V_{ud}$  are ellipses centered around the origin; shown is the blue thick contour line for  $\Delta V_{ud}$  from SAF and blue thin contour lines for half and twice that value. The outer vertical and horizontal lines show  $\Delta V_{ud}$  for neutron beta decay, using averages from [18]; their crossing point far from the origin indicates that PDG’s average is currently not competitive with SAF. The lines denoted “UCNTAU only” and “PERKEO III only” show the selected data set used in Fig. 1(a), which makes neutron beta decay competitive. Dashed straight lines show the impact of experiments under construction.

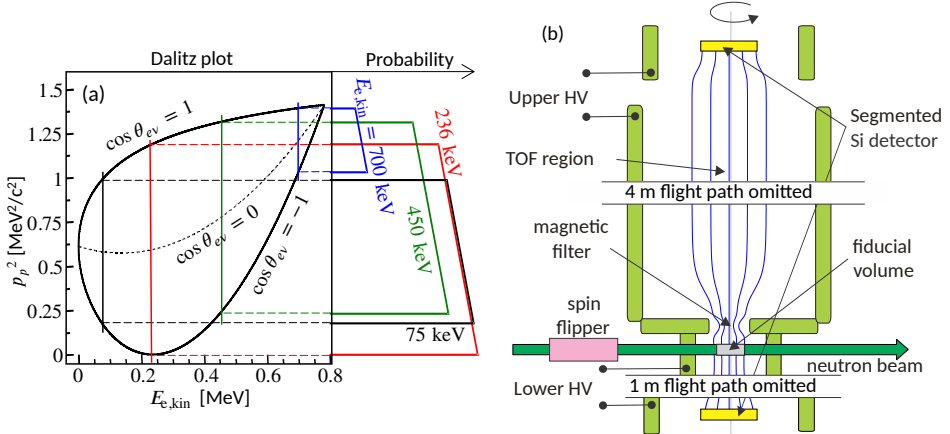
from the neutrino-electron correlation  $a$  ( $\lambda_a$ , see Refs. [26–29], with the SM analysis in the latter dominating the average), respectively. Unlike Fig. 1(b), Fig. 2 shows averages from many neutron beta decay experiments using the PDG prescription [18].

The goal of the Nab experiment [30–32] is to determine the  $a$  coefficient, and therefore  $\lambda$  with  $\delta\lambda/|\lambda| = 0.04\%$ , which is the dominant source of uncertainty in the determination of  $V_{ud}$  from Eq. (5). This measurement may also shed light on the disagreement between  $\lambda_A$  and  $\lambda_a$ . A natural extension of the Nab experiment will make use of the Nab spectrometer, but with a polarized neutron beam, to perform simultaneous measurements of the  $\beta$ -asymmetry and angular correlations involving polarized neutrons. This experiment, called pNab, will require only minor modification of the existing Nab apparatus, since the possibility to have highly polarized neutron beams and precise polarization analysis has been accommodated in the design of Nab. The pNab experiment would provide a new measurement of  $\lambda$  with a goal of  $\delta\lambda/|\lambda| = 0.02\%$  and new methods to control sources of systematic uncertainties through coincident detection of electrons and protons and ratios of spin-dependent observables. The Nab and pNab accuracy goals are illustrated with straight red dashed lines in Fig. 2. Note that further progress on the neutron lifetime only makes a substantial impact for the test of the CKM unitarity if it is accompanied with new results with Nab, pNab, or PERC [33, 34]. Besides neutron beta decay, there is progress in  $\Delta V_{ud}$  anticipated in the analysis of SAF [35]. There are also planned experiments studying beta decay in mirror nuclei [36, 37] and pions [38] that strive to achieve comparable accuracy in the CKM unitarity test.

## 2 Measurement principle of the Nab spectrometer

The determination of the neutrino electron correlation  $a$  in Nab relies on a measurement based estimate of the electron energy  $E_e$  and the proton momentum  $p_p$  for each neutron decay event. The electron momentum  $p_e$  is given by  $E_e$ , and so is the neutrino momentum  $p_\nu$  (in the infinite nuclear mass approximation). Momentum conservation ( $p_p^2 = p_e^2 + p_\nu^2 + 2p_e p_\nu \cos(\theta_{e\nu})$ ) and integration over the unobserved angle and spin allows one to transform the double differential decay rate from Eq. (3) into

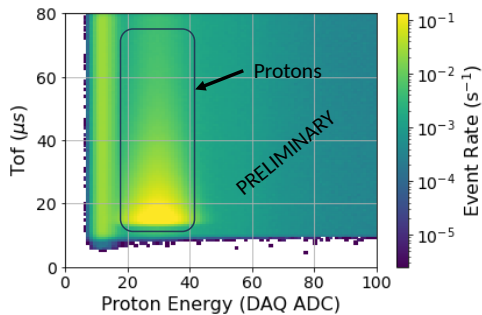
$$\frac{d\Gamma}{dE_e dp_p^2} \propto \rho(E_e) \begin{cases} 1 + a \frac{p_p^2 - p_e^2 - p_\nu^2}{2E_e E_\nu} + b \frac{m_e}{E_e} & (p_e - p_\nu)^2 \leq p_p^2 \leq (p_e + p_\nu)^2 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$



**Figure 3.** (a) Kinematically allowed region for neutron decays, bounded by the bold closed contour given by  $\cos \theta_{ev} = \pm 1$ . Probability distributions of  $p_p^2$  are shown alongside the right vertical axis for four representative electron kinetic energies  $E_{e,kin}$  following Eq. (6). (b) A sketch of the key Nab spectrometer components, not to scale. Magnetic field lines (blue), electrodes (light green), and coils (not shown) possess cylindrical symmetry around the vertical axis. The neutron beam, unpolarized for Nab, will be polarized for the follow-up pNab experiment.

Recoil order and radiative corrections need to be applied [39–41]. Fig. 3a shows the trapeziums described by Eq. (6) for multiple values of  $E_{e,kin} = E_e - m_e c^2$ . The  $a$  coefficient is determined from the slope of the  $p_p^2$  distribution for each electron energy  $E_{e,kin}$ , and the sharp edges help to understand the detector response for  $p_p^2$ .

Principles of the Nab spectrometer design and operation are illustrated in Fig. 3(b). The unpolarized cold neutron beam at the FNPB beamline [42] of the Spallation Neutron Source (SNS) passes through the spectrometer. A tiny fraction of neutrons decay in the fiducial decay volume. Decay protons have to pass through a magnetic filter above the fiducial volume followed by a 5 m flight path, to improve resolution. They are detected in the upper Si detector only, which is kept at a high voltage of  $-30$  kV to allow their detection. The Si detectors also accurately measure the energy of the decay electrons with keV-level resolution. Electron energy losses through backscattering of electrons are largely avoided thanks to the magnetic guide field that connects two Si detectors at both ends of the apparatus. Electrons might bounce, but are ultimately absorbed in the two detectors, whose signals are added. Only events with a total electron energy above a threshold of 100 keV are considered. Energy loss for detector dead-layer and bremsstrahlung have to be taken into account for electron energy extraction. It is addressed using in-situ calibrations using sets of monoenergetic lines from conversion electron sources and additional measurements in test facilities to fully characterize the energy response of the system. Above the magnetic filter, the proton momentum becomes parallel to the magnetic field as the field expands. The measured proton time of flight (TOF) gives an estimate of the proton momentum  $p_p$ : for a proton whose momentum is longitudinalized along the magnetic field, we have  $p_p \propto 1/t_p$ . Because we are actually measuring a time-of-flight difference between the proton and its corresponding electron, a small correction must be applied. Another correction is due to non-linearities in the mapping from  $t_p$  to  $p_p$  from the effect of the electric field close to the upper detector. These corrections are validated with an extensive GEANT4 simulation [43] of the electron and proton motion, followed by an extensive detector simulation [44].



**Figure 4.** Preliminary histogram of candidate events for electron-proton coincidences. The coincidence conditions include a temporal and position coincidence between two hits, and a threshold for the energy deposit of the first and second event. The histogram shows the energy and the TOF of the second event, presumed to be a proton. The vertical band at low proton energy is likely accidental background with electronic noise near the signal threshold.

### 3 Status of the experiment

The Nab experiment finished construction at the FNPB beamline at the end of 2021. Commissioning was held up for about a year when the large superconducting magnet (the largest cryogen-free superconducting magnet system) developed a thermal short that prevented energizing the superconducting coils. With the help of the manufacturer, Cryogenic Ltd., the problem was identified as a dislocated inner vacuum tube suspension element, and fixed. First commissioning data with both detector systems, taken in summer 2023, are reported below. Fig. 4 shows a distribution of protons as a function of their TOF after the electron arrival, and of deposited proton energy, recorded over 100 h. The TOF distribution is expected to peak at about  $15 \mu\text{s}$  with a sharp drop-off to lower values, and a gradual decline to higher values. Proton energy is centered around 30 ADC values in the figure, on top of accidental background. The shape of the peak confirms that electron-proton coincidences are correctly identified within the operational limits of the spectrometer. However, Nab commissioning has revealed new challenges that are being addressed during the current nine-month SNS facility outage. Most importantly, a significant portion of the detector pair pixels were inoperable which negatively affects both the data rate and electron energy reconstruction. The collaboration aims to transition to full production data taking in mid-2024. Two years of physics data taking are foreseen to reach the uncertainty goal for  $a$ , the  $e$ - $\nu$  correlation.

### 4 Outlook

Currently, there is possible evidence for a violation of CKM unitarity, based on data from superallowed nuclear decays, neutron and kaon decays. All inputs are under scrutiny. Neutron beta decay is free of nuclear structure uncertainties, but needs improved experimental precision. The Nab and pNab experiments share the goal of reducing the dominant uncertainty in the determination of  $V_{ud}$  with neutrons, i.e., measuring the value of  $\lambda$  independently, to sufficient precision. Given the current disagreement between results obtained by different measurement methods, a satisfactory gain in precision cannot be obtained by just improving the existing experiments: having the possibility to measure the  $a$  and  $A$  coefficients in the same apparatus promises to reveal whether the current discrepancy between the values of  $\lambda$  determined from each coefficient is real or not. A persistent discrepancy, reproducible by different measurement methods, would indicate a novel departure from SM predictions.

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